GCSE Maths – Algebra

Simultaneous Equations

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of simultaneous equations questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

Solve the following simultaneous equations by elimination:

$$5x + y = 27$$
$$x + 2y = 9$$

Step 1: Choose which variable to eliminate, then multiply one whole equation to ensure that the coefficient of the variable we are choosing to eliminate is the same in each equation.

For this example, we will eliminate the variable 'y'. To do this, we want to make sure that the coefficient is the same in each equation. Therefore, we will multiply the first equation by 2*.*

$$5x + y = 27 \quad \rightarrow (\times\, 2) \rightarrow \quad 10x + 2y = 54$$

Step 2: Add or subtract the two equations to eliminate the chosen variable.

We can subtract the second equation from the first to eliminate the 'y' variable entirely.

$$\begin{aligned} &(10x + 2y = 54) \\ -\ &(x + 2y = 9) \\ &\qquad\quad 9x = 45 \end{aligned}$$

Step 3: Solve for the remaining variable, then substitute the value of the known variable back into the original equations to find the value of the eliminated variable.

We can solve for x*:*

$$9x = 45$$
$$x = 5$$

Now substitute this x *value back into one of the equations:*

$$x + 2y = 9$$
$$5 + 2y = 9$$
$$2y = 4$$
$$y = 2$$

Be sure to check with both equations to avoid making any mistakes!

Indeed, $x = 5$, $y = 2$ *satisfies the other equation too:*

$$5x + y = 27$$
$$5(5) + 2 = 27$$

Guided Example

Solve the following simultaneous equations by elimination:

$$① \; 2x + 3y = 6$$
$$② \; x + y = 1$$

Step 1: Choose which variable to eliminate, then multiply one whole equation to ensure that the coefficient of the variable we are choosing to eliminate is the same in each equation.

Eliminate x so ② $\times 2$ to make coefficients equal.

$$x + y = 1 \xrightarrow[(\times 2)]{} 2x + 2y = 2 \; ③$$

this is equation 3.

Step 2: Add or subtract the two equations to eliminate the chosen variable.

$$① - ③ \rightarrow \begin{array}{r} (2x + 3y = 6) \\ -\;(2x + 2y = 2) \\ \hline 1y = 4 \end{array}$$

x is eliminated

$6 - 2 = 4$

$3y - 2y = 1y$

Step 3: Solve for the remaining variable, then substitute the value of the known variable back into the original equations to find the value of the eliminated variable.

$y = 4$ and $x + y = 1$ sub $y = 4$

$x + 4 = 1$

$-4 \quad x = -3 \quad -4$

$\boxed{x = -3, \; y = 4}$

check in the other equation:

$$2x + 3y = 6$$
$$2(-3) + 3(4) = 6$$
$$-6 + 12 = 6$$
$$6 = 6$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. Solve the following simultaneous equations.

a) $7a + 3b = 51$
$4a + b = 22 \xrightarrow{(\times 3)} 12a + 3b = 66$

eliminate b

$(12a + 3b = 66)$
$-\ (7a + 3b = 51)$
$5a = 15$

$\div 5\ \ 5a = 15$
$a = 3$

$4a + b = 22$
$4(3) + b = 22$
$-12\ \ 12 + b = 22$
$b = 10$

$a = 3$
$b = 10$

b) $5c - 4d = 29$
$c + 3d = 21 \xrightarrow{(\times 5)} 5c + 15d = 105$

eliminate c

$(5c + 15d = 105)$
$-\ (5c - 4d = 29)$
$19d = 76$

$\div 19\ \ 19d = 76$
$d = 4$

$c + 3d = 21$
$c + 3(4) = 21$
$-12\ \ c + 12 = 21$
$c = 9$

$c = 9$
$d = 4$

c) $3p - 5q = 16$
$q + p = -2 \xrightarrow{(\times 3)} 3q + 3p = -6$

eliminate p

$(3p - 5q = 16)$
$-\ (3p + 3q = -6)$
$-8q = 22$

$\div -8\ \ -8q = 22$
$q = -2.75$

$q + p = -2$
$(-2.75) + p = -2$
$+2.75\ \ p = 0.75$

$p = 0.75$
$q = -2.75$

d) $6x - 8y = 19$
$10x + 2y = 1 \xrightarrow{(\times 4)} 40x + 8y = 4$

eliminate y

$(40x + 8y = 4)$
$+\ (6x - 8y = 19)$
$46x = 23$

$-8y$ and $+8y$ have opposite signs, so add to eliminate.

$\div 46\ \ 46x = 23$
$x = 1/2$

$10x + 2y = 1$
$10(1/2) + 2y = 1$
$-5\ \ 5 + 2y = 1$
$2y = -4$
$\div 2\ \ y = -2$

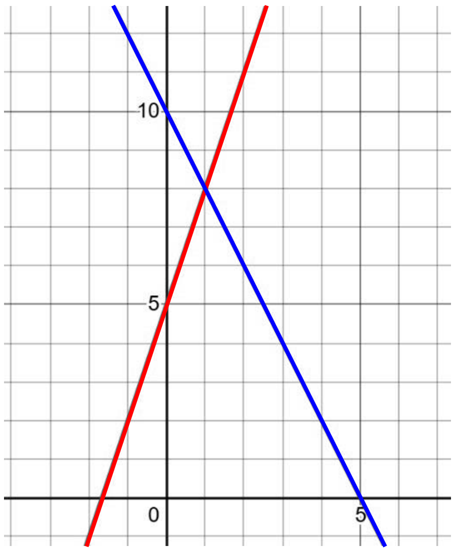
$x = 1/2$
$y = -2$

Section B

Worked Example

Use the graph below to find an approximate solution to the following simultaneous equations:

$$\boldsymbol{y = 3x + 5}$$
$$\boldsymbol{y = -2x + 10}$$



Step 1: Match the lines on the graph to the equations given.

The red line represents $y = 3x + 5$, which can be identified by the fact that the gradient is positive. Therefore, the blue line is $y = -2x + 10$.

Step 2: Identify the coordinates at which the lines intersect.

The point where the lines intersect represent the values of x and y that work for both equations, so it is the solution.

On this graph, the lines intersect at point $(1,8)$.

Step 3: Write out the solution to the simultaneous equations and substitute the values in to check.

$$x = 1$$
$$y = 8$$

Check by substitution:

$$y = 3x + 5$$
$$8 = 3(1) + 5$$

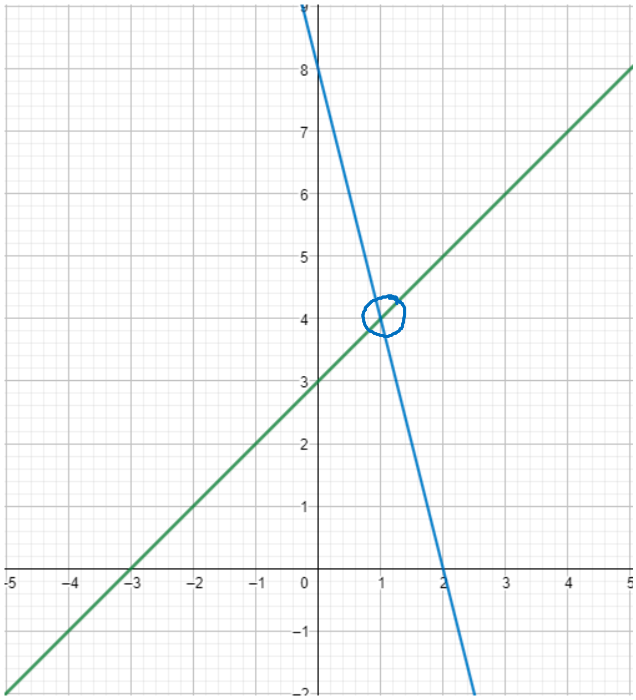
$$y = -2x + 10$$
$$8 = -2(1) + 10$$

Guided Example

Construct a graph to find the solutions to the following simultaneous equations:

$$y = x + 3$$
$$y = -4x + 8$$

Step 1: Construct the graph.



Step 2: Identify the coordinates at which the lines intersect.

The lines intersect at $(\underbrace{1}_{x=1}, \underbrace{4}_{y=4})$.

Step 3: Write out the solution to the simultaneous equations and substitute the values in to check.

$y = x + 3$
$(4) = (1) + 3$
$4 = 4$ ✓

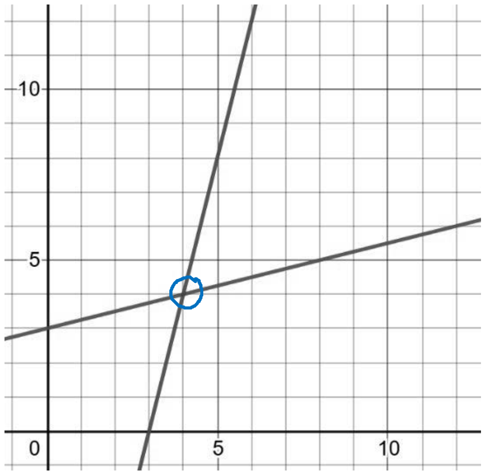
$y = -4x + 8$
$4 = -4(1) + 8$
$4 = -4 + 8$
$4 = 4$ ✓

Both equations are valid so:

$x = 1$
$y = 4$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

2. Use the graph to find approximate solutions to the following simultaneous equations:
$y = \frac{1}{4}x + 3$
$y = 4x - 12$

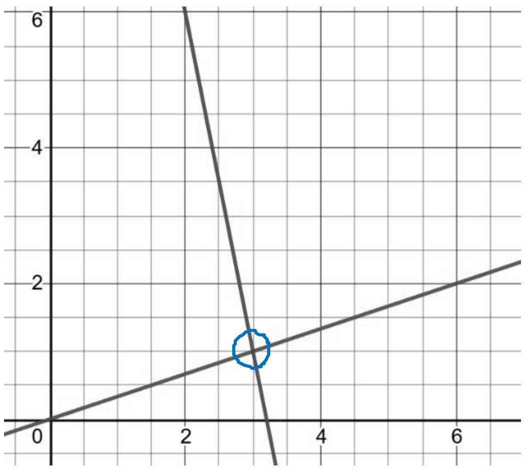


Intersect at (4, 4).

$y = 4x - 12$
$4 = 4(4) - 12$
$4 = 16 - 12$
$4 = 4$ ✓

$x = 4$
$y = 4$

3. Use the graph to find approximate solutions to the following simultaneous equations:
$y = x - 2y$
$y = -5x + 16$



Intersect at (3, 1).

$y = x - 2y$
$3y = x$
$3(1) = 3$
$3 = 3$ ✓

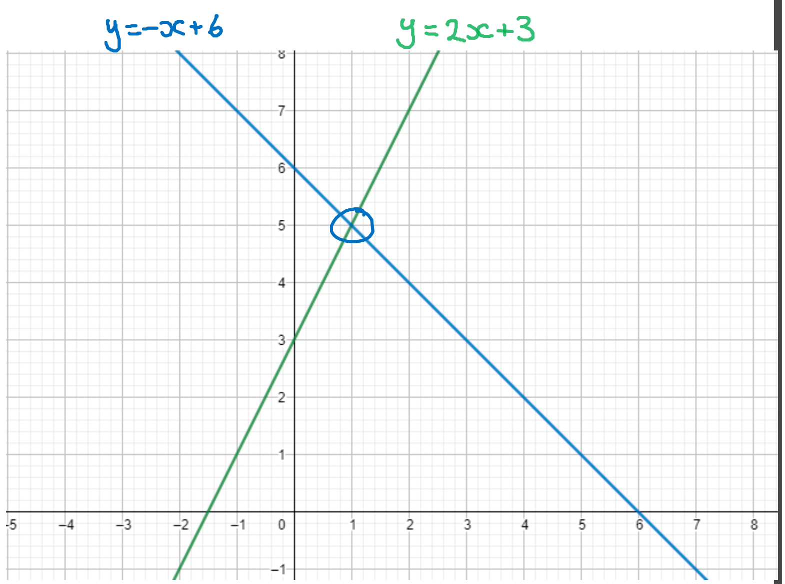
$x = 3$
$y = 1$

4. Construct a graph to find the approximate solutions to the simultaneous equations:

$y = 2x + 3$
$y = -x + 6$

$y = \underbrace{2}_{\text{gradient} = 2}x + \underbrace{3}_{y\text{-intercept} = 3}$

$y = \underbrace{-x}_{\text{gradient}=-1} + \underbrace{6}_{y\text{-intercept} = 6}$



Intersect at $(1,5)$.

$y = 2x + 3$
$(5) = 2(1) + 3$
$5 = 2 + 3 \rightarrow 5 = 5$ ✓

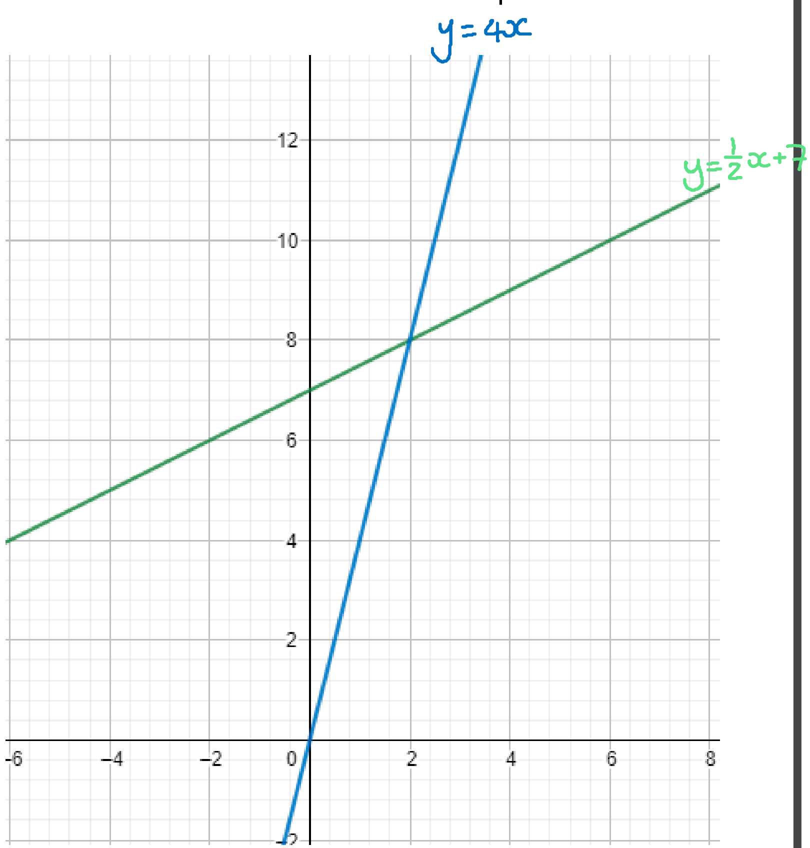
$x = 1$
$y = 5$

5. Construct a graph to find the approximate solutions to the simultaneous equations:

$y = \frac{1}{2}x + 7$
$y = 4x$

$y = \underbrace{\tfrac{1}{2}}_{\text{gradient} = \frac{1}{2}}x + \underbrace{7}_{y\text{-intercept} = 7}$

$y = \underbrace{4}_{\text{gradient} = 4}x$ $\quad$ y-intercept $= 0$



Intersect at $(2,8)$.

$y = 4x$
$8 = 4(2)$
$8 = 8$ ✓

$x = 2$
$y = 8$

Section C – Higher Only

Worked Example

Solve the following simultaneous equations by substitution:

$$\boldsymbol{y + x = 10}$$
$$\boldsymbol{x^2 + 4 = y + 6}$$

Step 1: Rearrange the linear equation to obtain one of the unknowns on its own.

The linear equation is the one that does not contain any squared, cubed, or higher order terms – every variable is to the power of 1 or a constant. In this case the linear equation is $y + x = 10$ *since the other equation contains an* x^2 *term.*
Rearranging to obtain one of the unknowns on its own:

$$y = 10 - x$$

Step 2: Substitute the linear equation into the quadratic equation, then solve the quadratic equation.

We substitute the linear equation as y *in the quadratic equation:*

$$x^2 + 4 = \boldsymbol{y} + 6$$
$$\boldsymbol{y} = 10 - x$$

$$x^2 + 4 = 10 - x + 6$$

Simplify the equation:

$$x^2 + x - 12 = 0$$

Solve the quadratic equation by factorisation or by using the quadratic formula:

$$x^2 + x - 12 = 0$$
$$(x - 3)(x + 4) = 0$$

The solutions are $\boldsymbol{x = 3}$ *and* $\boldsymbol{x = -4}$.

Step 3: Substitute the two solutions for x into the original linear equation to find the corresponding y values for each variable.

Substitute in $x = 3$*:*

$$y = 10 - x$$
$$y = 10 - 3$$
$$y = 7$$

Substitute in $x = -4$*:*

$$y = 10 - x$$
$$y = 10 - (-4)$$
$$y = 14$$

Guided Example

Solve the following simultaneous equations by substitution:

$$x^2 + 10 = y + 9$$
$$y = x + 7$$

Step 1: Rearrange the linear equation to obtain one of the unknowns on its own.

$y = x + 7$ ← this is already in the right format

Step 2: Substitute the linear equation into the quadratic equation, then solve the quadratic equation.

sub in $y = x+7$

$$x^2 + 10 = (x+7) + 9$$

$$x^2 + 10 = x + 16$$

$-x, -16$

$$x^2 - x - 6 = 0$$

$-x, -16$

$(x-3)(x+2) = 0$ ← factorise, complete the square or use the quadratic formula to find solutions.

$x = 3$

$x = -2$

Step 3: Substitute the solutions for x into the original linear equation to find the corresponding values for each variable.

$x = 3$	$x = -2$
$y = x + 7$	$y = x + 7$
$y = (3) + 7$	$y = (-2) + 7$
$y = 10$	$y = 5$

pair up solutions

$x = 3$, $y = 10$ $\qquad$ $x = -2$, $y = 5$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

6. Solve the following simultaneous equations by substitution:

a) $y + x = 3 \rightarrow y = 3 - x$
$y = x^2 - 2x + 1$

make y the subject

Substitute $3-x$ for y

$(3-x) = x^2 - 2x + 1$ $\quad +x, -3$
$0 = x^2 - x - 2$
$0 = (x-2)(x+1)$ factorise
$x = 2$
$x = -1$

$x = 2$
$y = 3 - x$
$y = 3 - 2$
$y = 1$

$x = 2$
$y = 1$

$x = -1$
$y = 3 - (-1)$
$y = 4$

$x = -1$
$y = 4$

b) $x - 6 = y \rightarrow x = y + 6$
$x + y^2 = 12$

$(y + 6) + y^2 = 12$
$y^2 + y + 6 = 12$ $\quad -12$
$y^2 + y - 6 = 0$
$(y-2)(y+3) = 0$
$y = 2$
$y = -3$

$y = 2$
$x = y + 6$
$x = (2) + 6$
$x = 8$

$x = 8$
$y = 2$

$y = -3$
$x = (-3) + 6$
$x = 3$

$x = 3$
$y = -3$

c) $2x + 4y = 14$
$x^2 + 3y - x = 27$

$2x + 4y = 14$ $\quad -2x$
$4y = 14 - 2x$ $\quad \div 4$
$y = \frac{7}{2} - \frac{1}{2}x$

$x^2 + 3\left(\frac{7}{2} - \frac{1}{2}x\right) - x = 27$
$x^2 + \frac{21}{2} - \frac{3}{2}x - x = 27$ $\quad -27$
$x^2 - \frac{5}{2}x - \frac{33}{2} = 0$
×2 to remove fraction
$2x^2 - 5x - 33 = 0$
$(2x - 11)(x + 3) = 0$
$x = \frac{11}{2}$ $\quad x = -3$

$x = \frac{11}{2}$
$y = \frac{14}{4} - \frac{1}{2}x$
$y = \frac{14}{4} - \frac{1}{2}\left(\frac{11}{2}\right)$
$y = \frac{3}{4}$

$x = \frac{11}{2}$
$y = \frac{3}{4}$

$x = -3$
$y = \frac{14}{4} - \frac{1}{2}(-3)$
$y = 5$

$x = -3$
$y = 5$

d) $7x + 10 = y - 1$
$y^2 - x = 4y + 1$

multiply whole equation by 7 to remove the fraction.

$7x = y - 11$ $\quad \div 7$
$x = \frac{y - 11}{7}$

$y^2 - \left(\frac{y-11}{7}\right) = 4y + 1$ $\quad \times 7$
$7y^2 - y + 11 = 28y + 7$ $\quad -28y, -7$
$7y^2 - 29y + 4 = 0$
$(7y - 1)(y - 4) = 0$
$y = \frac{1}{7}, \; y = 4$

$y = \frac{1}{7}$
$x = \frac{y-11}{7}$
$x = \frac{\left(\frac{1}{7}\right) - 11}{7}$ $\quad \div 7$
$x = -\frac{76}{49}$

$x = -\frac{76}{49}$
$y = \frac{1}{7}$

$y = 4$
$7x = 4 - 11$
$7x = -7$ $\quad \div 7$
$x = -1$

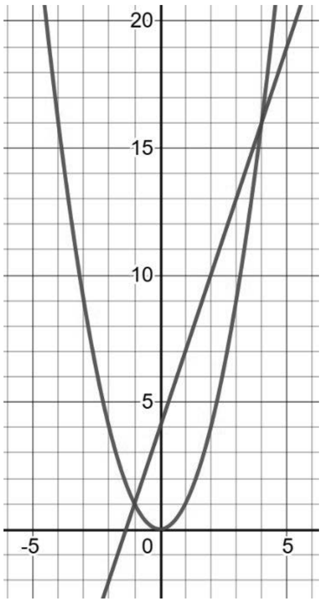
$x = -1$
$y = 4$

Section D – Higher Only

Worked Example

Use the graph to find the solutions to the following simultaneous equations.

$$\boldsymbol{y = x^2}$$
$$\boldsymbol{y = 3x + 4}$$



Step 1: Match the curves on the graph to the equations given.

The red line represents $y = x^2$ and the blue line represents $y = 3x + 4$. This is clear by the fact that the blue line must clearly match the equation of a straight line which is of the form $y = mx + c$.

Step 2: Identify the coordinates at which the lines intersect.

The points at which the lines intersect represent the values of x and y that satisfy both equations, so they are the solutions.

The lines intersect at $(-1, 1)$ and $(4, 16)$. This means that the solutions are:

$$\boldsymbol{x = -1, y = 1,}$$
and
$$\boldsymbol{x = 4, y = 16.}$$

Step 3: Substitute the values into the equations to check.

Substitute in $x = -1$:

$$y = x^2 = (-1)^2 = 1$$
$$y = 3x + 4 = 3(-1) + 4 = 1$$

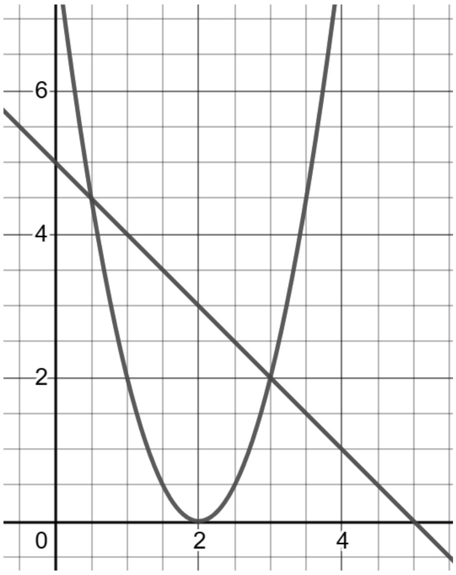
Substitute in $x = 4$:

$$y = x^2 = (4)^2 = 16$$
$$y = 3x + 4 = 3(4) + 4 = 16$$

Guided Example

Use the graph to find the solutions to the following simultaneous equations.

$$y = 2(x-2)^2$$
$$y = -x + 5$$



Step 1: Match the lines on the graph to the equations given.

contains a squared² term

Curve is a quadratic → $y = 2(x-2)^2$

Straight line is linear → $y = -x+5$

Step 2: Identify the coordinates at which the lines intersect.

Intersect at $(3, 2)$ and $\left(\frac{1}{2}, \frac{9}{2}\right)$.

$x=3, y=2$ $\quad x=\frac{1}{2}, y=\frac{9}{2}$

Step 3: Substitute the values in to check.

when $x=3, y=2$:

$y=-x+5$

$(2) = -(3)+5$

$2 = 2$ ✓

when $x=\frac{1}{2}, y=\frac{9}{2}$:

$y=-x+5$

$\left(\frac{9}{2}\right) = -\left(\frac{1}{2}\right)+5$

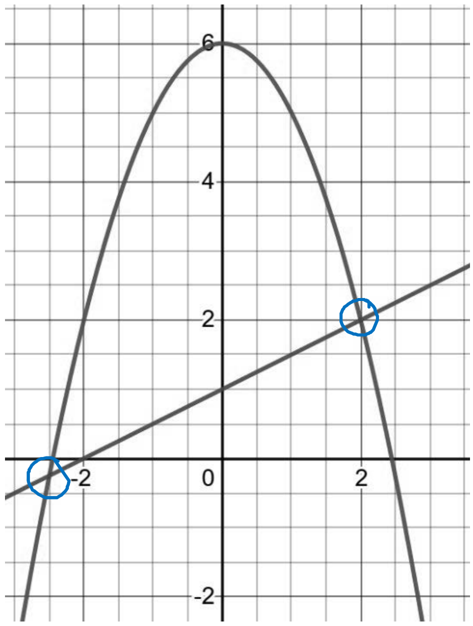
$\frac{9}{2} = \frac{9}{2}$ ✓

$x = 3$
$y = 2$

$x = \frac{1}{2}$
$y = \frac{9}{2}$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

7. Use the graph to find approximate solutions to the following simultaneous equations:
$y = -x^2 + 6$
$y = \frac{1}{2}x + 1$



Intersect at $(2,2)$ and $\left(-\frac{5}{2}, -\frac{1}{4}\right)$.

when $x=2$:
$y = -(2)^2 + 6$
$y = -4 + 6$
$y = 2$

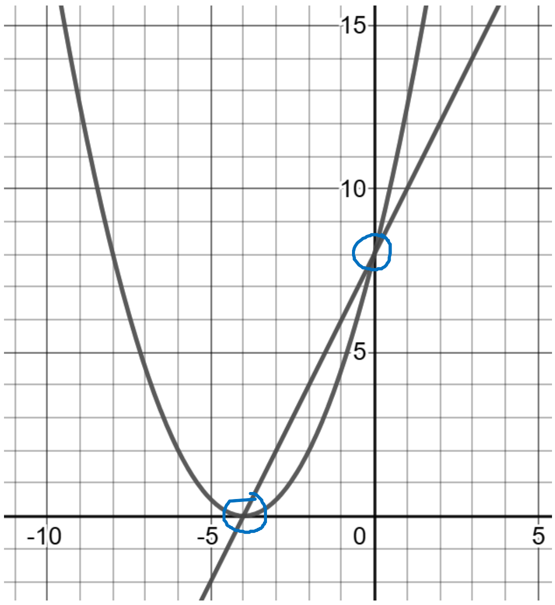
$x = 2$
$y = 2$

when $x = -\frac{5}{2}$:
$y = -\left(-\frac{5}{2}\right)^2 + 6$
$y = -\frac{25}{4} + 6$
$y = -\frac{1}{4}$

$x = -\frac{5}{2}$
$y = -\frac{1}{4}$

8. Use the graph to find approximate solutions to the following simultaneous equations:
$y = \frac{1}{2}(x+4)^2$

$y = 2x + 8$



Intersect at $(0,8)$ and $(-4,0)$.

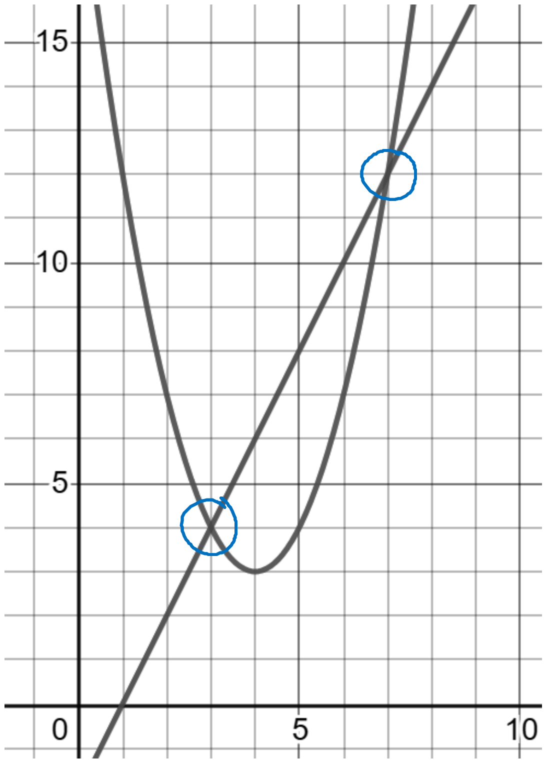
when $x=0$:
$y = 2(0) + 8$
$y = 8$

$x = 0$
$y = 8$

when $x=-4$:
$y = 2(-4) + 8$
$y = -8 + 8$
$y = 0$

$x = -4$
$y = 0$

9. Use the graph to find approximate solutions to the following simultaneous equations:
$y = (x-4)^2 + 3$
$y = 2x - 2$



Intersect at (7, 12) and (3, 4).

when $x = 7$:

$y = 2(7) - 2$
$y = 14 - 2$
$y = 12$ ✓

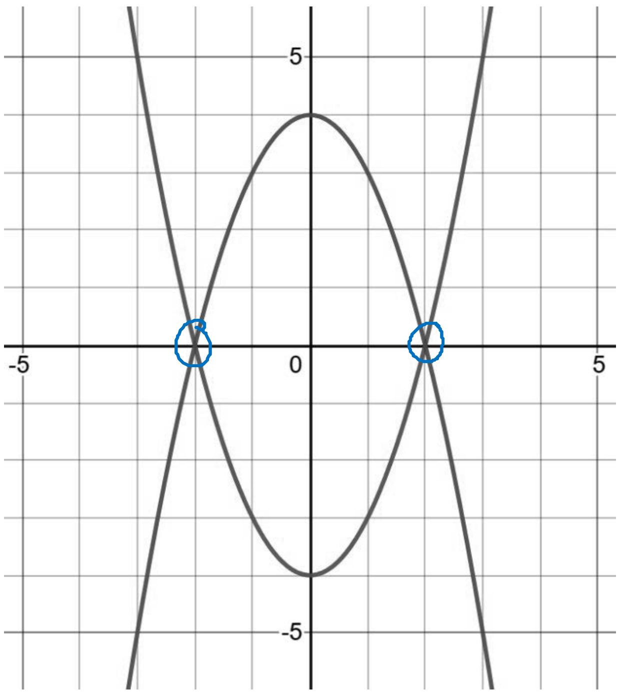
$x = 7$
$y = 12$

when $x = 3$:

$y = 2(3) - 2$
$y = 6 - 2$
$y = 4$ ✓

$x = 3$
$y = 4$

10. Use the graph to find approximate solutions to the following simultaneous equations:
$y = x^2 - 4$
$y = -x^2 + 4$



Intersect at (2, 0) and (-2, 0).

when $x = 2$:

$y = 2^2 - 4$
$y = 4 - 4$
$y = 0$ ✓

$x = 2$
$y = 0$

when $x = -2$:

$y = (-2)^2 - 4$
$y = 4 - 4$
$y = 0$ ✓

$x = -2$
$y = 0$